

Math 266.3 Midterm — Oct 24—2003

1. Determine an equation of the quadratic polynomial which passes through the points $(-1, 1)$, $(1, -5)$, and $(2, -2)$. $n=3 \quad Ax^2+Bx+C=0$

2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, then $6 = 1 + 1 - 3 - 1 - 2$

~~Find the trace $\text{tr}(A)$ of $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.~~

~~Find the determinant of A , $\det(A)$, by using the cofactor expansion in the third column.~~

~~Find A^2 .~~

~~Use the method of Gauss-Jordan elimination to find A^{-1} . Use an augmented matrix, and indicate which row operations are being performed at each step.~~

~~Encode the message "UPTAKE" using the coding matrix A .~~

~~The message 13,5,28,16 was encoded using the coding matrix $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Decode it.~~

3. (a) Find the angle between the vectors $(1, 2, -2)$ and $(3, -1, 1)$.

~~(b) Find and sketch the image of the triangle with vertices $(-1, 1)$, $(1, 1)$, $(0, 2)$ under the mapping $T(x, y) = (x + y + 1, y - 2)$.~~

4. Prove that a transformation defined by a 2×2 nonsingular matrix always maps straight lines into straight lines.

THE END

University of Saskatchewan
Department of Mathematics & Statistics

Instructor: M. Marshall

MATH 266.3 (02)

Time 50 minutes

Term Test 3

April 3, 2002

Marks:

- [6] 1. Let A be an $m \times n$ matrix.
(a) Define the row space of A , the column space of A and the nullspace of A .
(b) Define the rank of A and the nullity of A and describe the relationship between the two.
- [6] 2. Let $A = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 3 & 4 & 5 \end{pmatrix}$.
(a) Determine the reduced row echelon form U of the matrix A .
(b) Describe the relationship between the columns of A . In particular, determine a basis for the column space of A .
(c) Determine the nullspace of A and a basis for the nullspace of A .
- [7] 3. Let V and W be vector spaces.
(a) Define what it means to say that a function $L : V \rightarrow W$ is a linear transformation.
(b) Define the kernel of a linear transformation $L : V \rightarrow W$.
(c) Define $L(S)$ where $L : V \rightarrow W$ is a linear transformation and S is a subspace of V .
(d) Prove that if $L : V \rightarrow W$ is a linear transformation and S is a subspace of V then $L(S)$ is a subspace of W .
- [5] 4. Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation defined by $L(x) = (x_1, x_1 - x_2)^T$.
(a) Determine the matrix of L with respect to the standard ordered basis $[e_1, e_2]$ of \mathbb{R}^2 .
(b) Determine the matrix of L with respect to the ordered basis $[v_1, v_2]$ of \mathbb{R}^2 where $v_1 = (1, 1)^T$, $v_2 = (1, -1)^T$.
(c) Determine an invertible 2×2 matrix S such that $B = S^{-1}AS$ where A is the matrix from part (a) and B is the matrix from part (b).

[24] Total

** The End **

University of Saskatchewan
Department of Mathematics & Statistics

Instructor: M. Marshall
Time 50 minutes

MATH 266.3 (02)
Term Test 2

March 6, 2002

Marks:

- [3] 1. For what values of b is the matrix $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & b & -2 \\ b & 1 & 3b \end{pmatrix}$ non-singular?
- [3] 2. Use the Wronskian Test to decide if the functions $1, x, x^2, x^3$ are linearly independent.
- [6] 3. Let $S = \{(2a + b, a, b)^T \mid a, b \in \mathbb{R}\}$.
- (a) Prove that S is a subspace of the vector space \mathbb{R}^3 .
- (b) Determine a basis for S . Justify your answer.
- [6] 4. Let $\underline{v}_1 = (1, 0, -1)^T$, $\underline{v}_2 = (2, 1, 4)^T$, $\underline{v}_3 = (7, 2, 5)^T$.
- (a) Express \underline{v}_3 as a linear combination of \underline{v}_1 and \underline{v}_2 .
- (b) Describe $\text{Span}(\underline{v}_1, \underline{v}_2, \underline{v}_3)$ geometrically.
- [6] 5. Let A be a fixed $m \times n$ matrix.
- (a) Define the nullspace $N(A)$ of A .
- (b) Prove that $N(A)$ is a subspace of \mathbb{R}^n .

[24] Total

** The End **

Marks:

- [6] 1. (a) State the definition of a subspace of a vector space.
(b) Verify that the set S consisting of all polynomials f of degree less than 3 which satisfy $f(2) = 0$ is a subspace of the vector space P_3 .
(c) Determine a basis for the vector space S in part (b). Explain.
- [6] 2. Suppose that V is a vector space and that $\underline{v}_1, \dots, \underline{v}_k$ are elements of V . As usual, $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$ denotes the set of all linear combinations of $\underline{v}_1, \dots, \underline{v}_k$.
(a) Prove that $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$ is a subspace of V .
(b) Prove that if $\underline{v}_1, \dots, \underline{v}_k$ are linearly independent then each vector \underline{v} in $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$ is expressible uniquely as a linear combination of $\underline{v}_1, \dots, \underline{v}_k$.
- [6] 3. Let $A = \begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & 5 \end{pmatrix}$.
(a) Determine the rank and nullity of A . Explain.
(b) Determine a basis for the column space of A . Explain.
(c) Determine a basis for the null space of A . Explain.
- [6] 4. Consider the ordered bases $[\underline{u}_1, \underline{u}_2]$ and $[\underline{v}_1, \underline{v}_2]$ of \mathbb{R}^2 defined by $\underline{u}_1 = (1, 1)^T$, $\underline{u}_2 = (1, 2)^T$, $\underline{v}_1 = (0, 1)^T$, $\underline{v}_2 = (1, 2)^T$.
(a) Determine the transition matrix from $[\underline{u}_1, \underline{u}_2]$ to $[\underline{v}_1, \underline{v}_2]$.
(b) Express $5\underline{u}_1 - 3\underline{u}_2$ as a linear combination of $\underline{v}_1, \underline{v}_2$.

[24] Total

** The End **

$$\begin{matrix} 5 & -2 \\ 5 & -2 \end{matrix} \quad \begin{matrix} 2 & 1 & 0 \\ 1 & 1 & 5 \end{matrix} \quad \begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix} \quad \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix}$$

Marks:

[6]

1. Consider the system of linear equations:

$$x_1 - x_2 = 3$$

$$2x_1 - 2x_2 + x_3 + x_4 = 8$$

$$x_1 - x_2 + x_3 + x_4 = 5$$

- (a) Write a single matrix equation which is equivalent to this linear system.
(b) Determine the general solution of this linear system using the method of Gaussian elimination. Show your work.

[8]

2. Let $B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -1 & 3 & 5 \end{pmatrix}$.

- (a) Determine elementary matrices E_1, E_2, E_3 such that $E_3 E_2 E_1 B$ is upper triangular.
(b) Express B in the form $B = LU$ where L is lower triangular and U is upper triangular.

[4]

3. Let $\underline{b}_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $\underline{b}_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, $\underline{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and let $\underline{e}_1, \underline{e}_2, \underline{e}_3$ denote the standard basis of \mathbb{R}^3 .

- (a) Express \underline{b} as a linear combination of $\underline{e}_1, \underline{e}_2$ and \underline{e}_3 .
(b) Is \underline{b} expressible as a linear combination of \underline{b}_1 and \underline{b}_2 ? Explain.

[6]

4. Prove that if A is any $m \times n$ matrix, B is any $n \times r$ matrix, and α is any scalar, then

$$\alpha(AB) = (\alpha A)B = A(\alpha B).$$

[24] Total

** The End **

University of Saskatchewan

[S. Kuhlmann]

MATHEMATICS 266.3

Time: 50 minutes

SECOND MIDTERM TEST

March 11, 2005

CLOSED BOOK, NO NOTES.

Show your work. Justify your answers.

- 1) a) Define the notion of an **invertible matrix**.
b) Prove that if A is an invertible $n \times n$ matrix, then for any each $n \times 1$ (column matrix) \mathbf{b} , the system of equations

$$A\mathbf{x} = \mathbf{b}$$

has exactly one solution.

- c) Solve the system

$$\begin{aligned} 3x_1 + 5x_2 &= b_1 \\ x_1 + 2x_2 &= b_2 \end{aligned}$$

by inverting the coefficient matrix.

- 2) a) Let A be an $n \times n$ matrix. Define a **signed elementary product from A** and the **determinant** of A .
b) Find a 3×3 triangular matrix D that satisfies that $\det(2D) = 16$.
c) Evaluate the determinant by reducing the matrix to row echelon form:

$$A = \begin{pmatrix} 4 & 2 & 6 & 2 \\ 2 & 0 & 2 & 2 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 6 \end{pmatrix}$$

- 3) a) Let V be a vector space and $S \subseteq V$. Define the **span** of S .
b) Prove that the span of S is a subspace of V .
c) Let $V = M_{2 \times 2}$ the vector space of 2×2 matrices. Find 4 matrices that span V .

**** The End ****